TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 5   2022

Section 1   Calculator Free
Matrices

STUDENT'S NAME   _____

DATE: Tuesday 30th August   **TIME:** 25 minutes   **MARKS**: 24

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

   Consider the matrices $A = \begin{bmatrix} 1 & 2 \\ -1 & k \end{bmatrix}$ and $B = \begin{bmatrix} k & 1 \\ 0 & -2 \end{bmatrix}$ where $k \in \mathbb{R}$.

   (a) Suppose $det(A) = 0$. Explain what this means. [1]

   (b) Determine the value(s) of k given that:

      (i) the matrix AB has no inverse. [3]

      (ii) $B^{-1} = \frac{1}{4}B$ [3]

2. (4 marks)

Determine the transformed equation of the line $2y = 3x + 4$, when it is transformed by the matrix $\begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$.

3. (4 marks)

Determine matrix **P** given that $\mathbf{AP} + \mathbf{P} = \mathbf{Q} + \mathbf{BP}$ and $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, $\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 3 & -6 \end{bmatrix}$.

4.   (9 marks)

(a)   If $\tan \theta = \dfrac{1}{2}$, determine the value of $\tan 2\theta$.   [1]

(b)   Determine the transformation matrix for a reflection in the line $y = \dfrac{x}{2}$.   [3]

(c)   When reflected in $y = \dfrac{x}{2}$, the image of the point $(a, -5)$ is $(2, b)$. Determine the value of a and the value of b.   [4]

TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 5   2022

Section 2   Calculator Assumed
Matrices

STUDENT'S NAME   _____

DATE: Tuesday 30th August   **TIME:** 20 minutes   **MARKS**: 21

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser, approved Formula sheet
Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5.   (10 marks)

Transformation Q is an anti-clockwise rotation about the origin of $\frac{4\pi}{3}$.

(a)   Represent transformation Q as a 2×2 matrix using exact values.   [2]

A second Transformation Matrix P is given by $P = \begin{bmatrix} 2 & 0 \\ 0 & 0.25 \end{bmatrix}$

(b)   Describe the transformation represented by the matrix P.   [2]

(c)   The triangle $A(1, 2)$, $B(-1, 4)$, $C(-6, 8)$ is transformed by the matrix Q followed by the matrix P to the triangle $A''B''C''$. Find coordinates of A'', B'' and C'', correct to 2 decimal places.   [2]

(d)   Determine the area of triangle $A''B''C''$ [2]

(e)   Determine the single matrix M that will transform the triangle $A''B''C''$ back to the triangle ABC using exact values. [2]

6. (7 marks)

A system of equations is given by
$$4x + ay - 9 = 0$$
$$-2x + 3y + 3 = 0$$

(a) Let the constant $a = -5$.

 (i) Express the system in matrix form $AX = B$.

[2]

 (ii) Determine A^{-1} and demonstrate use of matrix algebra to solve the system for X.

[3]

(b) Determine the value of a for which the system has no solution and comment on the relationship between the two lines that form the system when a has this value. [2]

7.   (4 marks)

Consider matrix $P = \begin{bmatrix} p & 3p-1 \\ 1 & p \end{bmatrix}$. Determine the value(s) of p such that $\det(\mathbf{P}) = \det(\mathbf{P^{-1}})$